A Critical Study of the Coherence between EBMs and ABMs in the Simulation of the Hawks-Doves-LawAbiders Society

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Abstract. We are interested in using Equation-based Models (EBMs) as formal specifications of macro-level features of social simulations, so that they could support the subsequent development of more detailed Agent-based Models (ABMs). In this work, we tried to establish the way a particular macro-level feature present only in the ABM of a certain society, namely, the density of the population in its environment, may influence the coherence between the results of the two models about the evolution of the population profile. We took as case study the HDL (Hawks, Doves, and LawAbiders) society. In our simulations, higher values of population density tend to produce less coherence between the two models, because of the interference between the population density parameter and the particular way the agents were programmed to wander in the environment. The work indicates the possibility of a systematic study of the interdependencies of ABMs and EBMs when they are used complementarily.

Keywords: Social simulation, Equation Based Models, Agent Based Models

1 Introduction

Social simulations may be addressed on at least two levels: the macro social level, which focuses on the global behavior of the society, and the micro social level, which *additionally* studies the local behavior of the individuals of the population.

The macro social level views the society as a whole, focuses on populations and sub-populations, with the predominance of the global view of the system, usually assuming that individuals are similar, with the same scale of values and even same behavior.

The micro social level focuses on individuals, their interactions, actions and behaviors, how they are born and die, and how macro level features are generated from (*emerge* from) the mass of individual interactions.

In this work, we make use of Equation Based Models (EBMs) to model directly the macro-level, while we use Agent Based Models (ABMs) to model *directly* the micro-level, and *indirectly* the macro-level via emergent features.

The problem then arises of the compatibility between the macro-level results produced by the two kinds of models. Equation Based Models (EBMs) make use of differential or difference equations [1, where the state of the system is represented by a set of numerical variables, and the future state of the system is determined from its current state through a set of difference or differential operators that model the dynamics of the system.

The Agent Based Models (ABM) work with the basic concept of an autonomous agent, which controls its own actions based on its perceptions of the environment, and that acts on that environment. Thus ABMs are composed of a set of agents that demonstrate individual behaviors, which collectively form the system.

ABMs and EBMs can be seen as rival models for social simulation [2]. Accordingly, the critical aspects that differentiate the two kinds of models are, among others:

- Individuals are characterized either separately (ABMs) or in an aggregate way (EBMs), which implicitly suggests that the best applicability of the ABMs is to the micro-level, while the best applicability of the EBMs is to the macro-level.
- In EBMs applied to the macro-level, the observables of the system concern aggregate information derived from the equation-driven evolution of aggregate information themselves.
- In ABMs applied to the micro-level, individuals interact with each other through their behavior in the environment, so that the observables of the system concern aggregate information derived from the log of individual behaviors.

The choice between one model or another, in the development of a social simulation, should be made, according to [2], in a case by case basis, centering around practical considerations (need to have control of behaviors of individuals in the simulation, need to take into account aspects of the interaction between individuals and the environment, etc.).

In this paper, we take the point of view that EBMs and ABMs can be treated as complementary, rather than rival models, so that the idea of choice between them becomes meaningless. We claim that if enough features can be identified that allow for a safe control of the degree of coherence between the macro-level results produced by the two kinds of models, than EBMs of the macro-level can be used as preliminary formal specifications of the ABMs that should be subsequently developed.

In such position as a formal specification of an ABM, an EBM can then be taken as a yardstick for the verification of the correctness of the definition and implementation of the ABM, possibly helping in the establishment of the faithfulness of the ABM (see, e.g., [3] for the problem of the faithfulness of social simulation models).

Such purpose requires, however, an assessment of the degree of coherence between the results that each kind of model may provide.

This paper focus on a particular issue, regarding that degree of coherence, namely, the impact that the density of the population of the ABM (that is, the number of agents per unit area in the environment) may have on the coherence of the macro-level results of the two models, in the cases where the casual spatial encounters between individuals are important for the functioning of the social system.

To have a concrete case study, we analyze the influence of the population density on the coherence between macro-level results provided by EBMs and ABMs of the HDL (Hawks-Doves-LawAbiders) kind of social system [1].

This article is presented as follows. Section 2 presents the problem studied for both models, the HDL society. In sections 3 and 4, are presented the EBM and ABM models of the HDL society, respectively. In Section 5 the results obtained with the simulations are compared, focusing on the issue of the evolution of the population profile. Section 6 presents the conclusions regarding this work.

2 The HDL Society

Martinez Coll describes the evolution of the society modeled here as the necessary evolution of the "Hobbesian state of nature", given a Bioeconomic perspective on it [4]. The "Hobbesian state of nature" is one in which members of the society are always competing for resources, so that, as the resources always belong to someone, conflicts between individuals arise all the time.

The members of this society are allowed to adopt one of three fighting strategies, respectively called the Hawks, Doves and LawAbiders strategies.

- The Doves never try to get resources from others, but expect them to leave voluntarily; a dove abandons its resource as soon as it is attacked; if two doves compete for the same resource, one of them wins by chance or persistence;
- The Hawks always attack and try to get resources from others; a hawk only gives up its action if it is badly hurt;
- LawAbiders never attack to get resources from others, but always defend themselves when attacked; a LawAbider may succeed or not in its defense, and thus may lose its resources or get new ones in each conflict.

During the evolution of the society, mechanisms such as inheritance, imitation and indoctrination may be used to spread the best strategy, so that the different strategies are adopted by the members of the society according to the degrees of success that the strategies are giving to their adopters in the conflicts.

Each strategy involves costs and gains. Any conflict between individuals with their respective strategies will result in a new balance of resources, following the rule of profit and loss governing that meeting. For example, when two hawks meet, they always fight and the cost of their fight is high. In general, when two individuals with the same strategy meet, both have the same chance of winning or losing resources.

3 Equation Based Model

The evolution of each strategy at time t, for the strategies $i = H, D, L$, of the respective sub-populations of Hawks, Doves and LawAbiders, is given [1] by the equation

$$P_i(t+1) = P_i(1 + F_i(t)) \tag{1}$$

where the size of the sub-population is represented by the percentage variables P_H, P_L and P_D, and the measure of fitness at time t by the variables $F_H(t)$, $F_D(t)$ and $F_L(t)$. The fitness of each strategy corresponds to how well that sub-population is doing in the environment, i.e., the degree of success of the strategy in relation to others.

The yield $Y_i(t)$ that a population i presents in a time t is given by the equation

$$Y_i(t) = \sum_j R_{ij} P_j(t) \tag{2}$$

where the gain R_{ij} is given by the rules of fight between an individual follower of the strategy i and an individual that follower the strategy j, as defined in Table 1.

The yield of the entire population of the system $Y_S(t)$ is obtained from the current profile of sub-populations

$$Y_S(t) = \sum_i Y_i(t) P_i(t) \tag{3}$$

Therefore, the fitness of the sub-population i at time t is given simply by the difference between its yield and the total yield of the population

$$F_i(t) = Y_i(t) - Y_S(t) \tag{4}$$

The higher the yield of a sub-population in relation of the others, the greater the probability of its strategy being imitated by members of the other sub-populations, or inherited by its descendants.

4 Agent Based Model

We developed an ABM[1] where individuals wander in the environment and the conflicts occur as they meet each other.

In each conflict, the result of the conflict is decided in accordance with the strategies adopted by the fighters. Following [1], the costs and rewards involved in the fights were defined as in Table 1:

[1] NetLogo was used for the development of the simulations studied in this work.

Table 1. The pay-off matrix of the conflicts in the HDL society

	Dove	Hawk	LawAbider
Dove	2,2	0,10	1,6
Hawk	10,0	-5,-5	2.5,-2.5
LawAbider	6,1	-2.5,2.5	5,5

Table 1 is obtained directly as a suitable intuitive numerical representation of the informal definition of the conflict rules ([4], cf. Sec. 2) and establishes a *bridge* between the macro-level EBM and the micro-level ABM, as it is *embedded both* in the procedure that calculates the results of the conflicts in the ABM, and in the R_{ij} parameters of the EBM equations.

Our ABM of the HDL society was implemented based on the information in Table 1. However, many additional non-EBM related decisions have had to be made, because at the micro-level several *particularities* of individuals (which are not explicitly indicated in the EBM) have be considered, specially, when such features interfere with technical issues relating to the *programming* of the agent behaviors.

This is an important issue because most of those micro-level particularities of the individuals often impact in strong ways the overall behavior of the multiagent society and, thus, the coherence between the macro-level results obtained through the ABM and those obtained through the EBM. In the following, we consider some of them.

At each step the individual chooses a random direction to move. But before that, it must observe if there are other individuals at the final destination. If there is no other agent in that point, the individual may move. If there is only one other agent, the individual moves to that point and a conflict happens. If the point that the individual has chosen to move to is already occupied by two agents, then we have decided to make it not move to that place, remaining in their place of origin, so that only conflicts between two individuals may occur at each position. The fact that an individual does not move to a place where there are already two other individuals is directly related to cost accounting, which is calculated for only two individuals. Clearly, this is an issue that is not taken into account in the EBM, since the environment is also not an issue for it (cf. [2]).

Also, the ABM has to define the spatial features of the individual moves in the environment. Those spatial features impact not only the probability of the occurrence of interactions between individuals, but also the scope of the *local observations* about the success of each strategy that each individual may make.

For instance, if the walking proceeds with steps that are too small, the size of the visited region in a given time may turn out to be too small compared with the overall size of the environment, possibly leading to problems like: a too limited sampling of the society, or an excessive repetitive number of conflicts with the same sub-group of individuals occupying a restricted part of the environment, etc., all leading to an insufficient, even plainly incorrect, assessment of the current yields that the various strategies are producing in the society of as a whole. This is clearly another problem in the development of the ABM that the EBM can not help to solve

In the ABM, another important aspect had to be defined independently of the EBM, which is the procedure by which each individual takes the decision to change or not to change the strategy it is currently adopting, in consequence of its perception of the current yields in the society. The EBM gives no hint on such strategy, it only specifies the resulting percentage of strategy changes that occur in the society at each moment, but not the procedures the individuals may adopt to make their private decisions.

In the ABM model, such decision procedures have to be made explicit. In the ABM implementation that we developed, each individual keeps a memory of the number of conflicts it has participated in, and of the balance of costs and rewards accumulated in those conflicts, and use either one of those information to determine when it is time to decide about changing strategy or not, and what such decision should be.

To choose the best strategy to change to, the individual has to accumulate various kinds of information. As examples, we have the number of individuals of each sub-population found in the past period under evaluation, in order to constitute a sample of the current distribution of individuals among the sub-populations of the society; also, the average balance of costs and rewards obtained by each such sample of sub-populations, so that an estimate of the yield of each sub-population can be calculated.

This calculation of such estimate of the yield of each sample sub-population was programmed as follows: at each conflict, the individual updates both the balance of costs and rewards and the number of individuals in the sample of the sub-population of the individual it has met. When it comes the time to consider a possible change of strategy, the individual proceeds as follows: it compares the yield of the sample of its own population to the total yield of the total sampled population. If the yield of the individual's own sampled sub-population is in disadvantage with respect to the yield of the total sampled population, the individual changes its strategy to that strategy which it has found with highest yield, in the sampled population; otherwise it keeps its strategy.

Alternative decision procedures had to be tested, in order to determine the best one, which is the one that best matches the rate of changes in the population profile, defined by the EBM in the parameter $1+F_i(t)$ that controls the variation of the size of the population P_i, in the equation (1). A criterion requiring an unconditional change to the best strategy at each decision moment gives a rate of change in the population profile that is usually too high, compared with that determined by the EBM. A criterion requiring a cautious decision, based for instance on a large difference between the yield of the individual's own sampled population and the yield of the overall sampled population, may lead to a too slow rate of change in the population profile, compared to those required by the EBM. Clearly, the EBM gives no direct hint on which decision procedure should be adopted by the individual agents of the ABM.

5 Results of Simulations

The initial set up for every simulation was defined with the following set of values: Hawks= 90%, Doves = 5%, LawAbiders = 5% of the overall population.

From such initial set up, a general picture of the behavior of the population can be given [1]. Initially, it is expected that the Hawks mainly conflict with each other, leading to high-costs to the decision to keep adopting the Hawks strategy. So, the Hawks gradually decide to adopt either the LawAbiders strategy or the Doves strategy. As the number of the doves stay small at the beginning, even with some Hawks becoming Doves, their sub-population tend to grow, since their is the best yields in such situation. But as the Doves population grows, their yield reduces, and Doves tend to become LawAbiders. As the number of LawAbiders increase, their yield increasingly becomes even better, producing a permanent tendency towards the spread of such strategy, the LawAbiders finally winning the competition.

We note, however, that the simulations were performed for three different simulation models: the EBM, the pure ABM (with agents strictly limited to a local view of the environment, as usually assumed in ABMs), and a so-called *multilevel* model [1] where agents were given access to *external oracles* that numerically account for certain features of the global environment. The following sub-sections compare the results obtained.

5.1. The EBM Simulations

The results of the EBM that we implemented faithfully meet the expectations, as shown in Fig. 1.

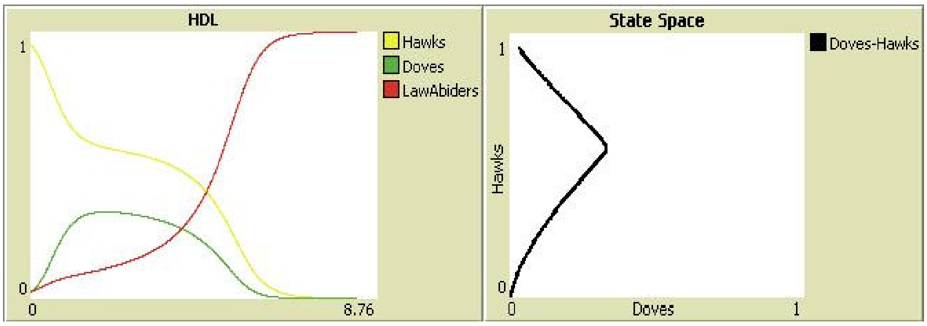

Fig. 1. Result of the EBM simulation.

5.2. The ABM Simulations

For the ABM simulations, the following agent parameters were determined: size of the steps at each simulation time equal to 5 environment cells; a number of 3 conflicts performed and assessed in order to decide about changing or not the strategy; and an environment of 50 to 50 cells, giving an environment area of 2500 cells.

In the case of the ABM simulations, we were specifically interested in the impacts that the density of the population of individuals in the environment would have on the faithfulness of the ABM simulations with respect to the EBM simulations.

The number of steps simulated is determined by the time that the variation of the population reaches some stability. These values were determined after a considerable number of simulations for each of the densities discussed in this work.

To assess such impact, we considered four different population densities, namely, population densities of 0.04, 0.2, 0.4 and 0.6 individuals / environment cell.

The simulations with the population density of 0.04 were considered qualitatively satisfactory after running about 500000 steps, resulting in population sizes of 88% of Law-Abiders, 7% of Hawks and 6% of Doves, compared to the combination of 100%, 0% and 0% required by the EBM. The average graph of the performed simulations can be seen in Fig. 2.

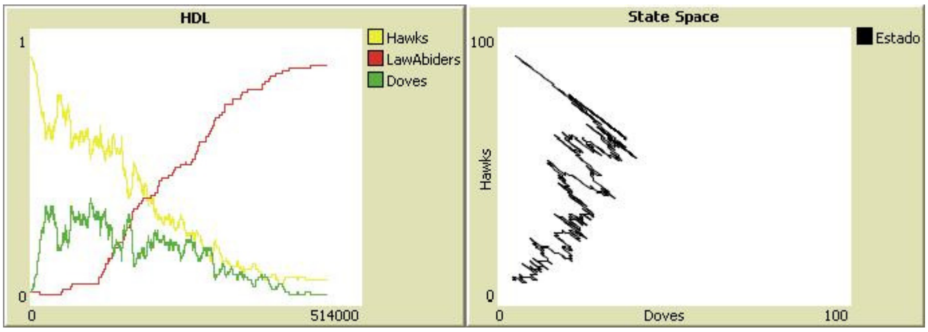

Fig. 2. ABM simulation with population-density = 0.04.

The simulations with population density of 0.4 were less satisfactory. The population sizes obtained were of 74% of LawAbiders, 18% of Hawks and 8% of Doves on the average, as shown in Fig. 3.

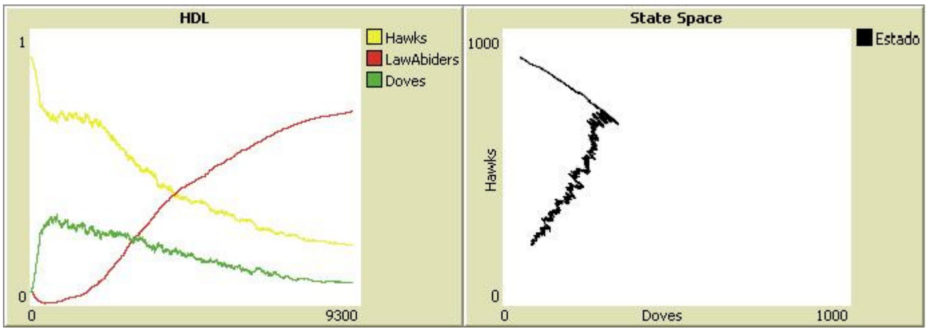

Fig. 3. ABM simulation with population-density = 0.4

The simulations with population density of 0.2 got worse results than simulations with population density of 0.04 and better than simulations with population density of 0.4. The profile obtained was 87,35% of LawAbiders, 8,55% of Hawks and 4,1% of Doves, as shown in Fig. 4.

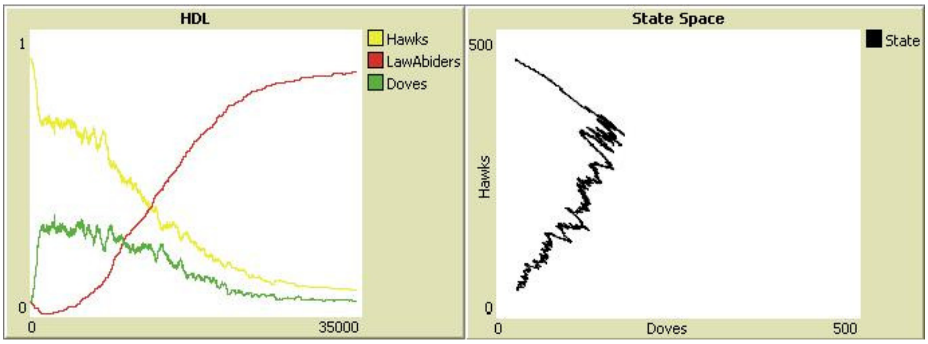

Fig. 4. ABM simulation with population-density = 0.2

The simulations with population density of 0.6 were the less satisfactory. The population sizes obtained were of 61,5% of LawAbiders, 26,5% of Hawks and 12% of Doves on the average, as shown in Fig. 5.

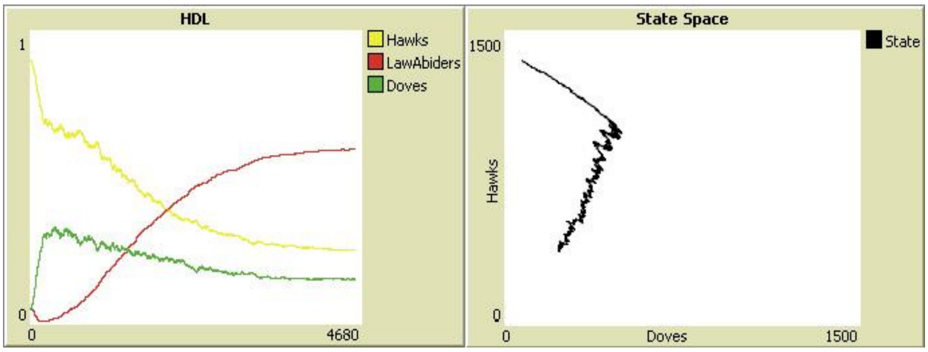

Fig. 5. ABM simulation with population-density = 0.6

We have found, then, that the ABM with the restriction that agents do not occupy cells with more then two individuals leads to the effect that the *higher the population density*, the *worse the faithfulness* of the ABM to the EBM.

This is seems to be so because with high population densities, the individuals have difficulty in finding places where to move to, thus tending to reduce the scope of their wandering and, so, tending to restrict their conflicts to their immediate neighbors, with the consequence that the quality of their assessments of the yields of the different sampled populations is reduced.

Similarly, in a population less dense, with fewer individuals per cell, the agents may take more time to find another agent to fight with, so retarding the decision on whether to change or not the strategy. As these individual decisions affect the total variation of the population at each moment, the final result takes longer to appear, thus requiring a larger number of steps to stabilize the number of individuals in each population. Also, retarding the decisions may lead them to be taken when they may be not suitable anymore, as the environment may have changed significantly during the sampling period.

Again, we think that this analysis shows clearly a strong influence on the result of the ABM simulation of an *unexpected interference* (from the sole EBM point of view) between a *neatly macro-level ABM parameter* (the population density) and a *neatly micro-level ABM parameter* (the decision criterion about where to move to at each instant).

The unexpectedness of the interference is even more strong as the programming of the criterion about how to wander in the environment would usually be treated as a "mere" programming implementation detail, assumed to have no impact on the social behaviors of the agents and on the overall functioning of the society.

5.3. The Simulations of the Multilevel Model

In a multilevel model [1], the simulations of individual behaviors are allowed to make use of information that has a global nature with respect to the society. The main advantage of multilevel models is the decrease of the need to assign to the very behaviors of the individual agents (forced to behave under restricted local views of the society) the responsibility of assessing the whole simulated society. Thus, the programming of the agents may concentrate on the particular behavioral issues that the simulation is designed to study.

Multilevel models could be imagined that make use of agent technologies in different ways, from the simple simulation of agent actions on the basis of the evolution of individualized numerical variables [1], to the full-fledge use of agent oriented programming techniques, where external oracles responsible for the global observation of the society communicate their findings to the agents through either messages or globally shared variables, an alternative that we have called *agent-based multilevel simulation* in [5].

The multilevel model we used in this study is of that agent-based kind. The individuals change strategy according to probabilities that depend of the values of the yields, which are calculated by external oracles to match the $1+F_i(t)$ factors in the EBM model.

The result of a simulation for a population density of 0.04 is shown in Fig. 6, where the steady point of the simulation is seen to have been achieved around 15000 simulation steps.

Clearly, the agent-based multilevel simulation performed better than any of the ABMs, due to the availability of better assessments of the yields of the sub-populations of the society.

For the sake of space, we let for a further paper the analysis of the interference of the variation of the population density on the results of this agent-base multilevel model.

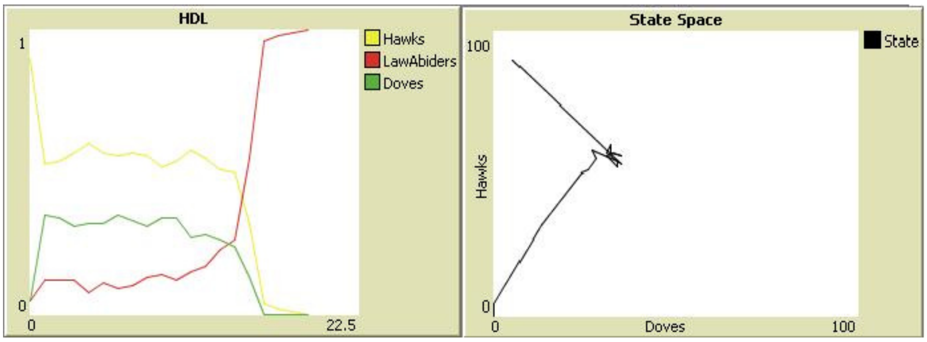

Fig. 6. Agent-base Multilevel simulation with population density = 0.04.

6 Conclusion

We initially targeted this study to the elucidation of the interference that the population-density could have on the faithfulness of the ABM with respect to the EBM.

We have found that the population-density, which is a macro-level parameter, has a strong connection to the micro-level rule of spatial displacement of the individuals in the ABM, to the point that the population-density parameter is able to jeopardize the quality of the results of the ABM simulations, if the displacement rule presents certain restrictions.

The restriction that agents do not occupy cells with more then two individuals, in the ABM simulations, leads to the effect that the higher the population density, the worse the faithfulness of the ABM to the EBM. This seems to be so because, in high population densities, the individuals have difficulty in finding places where to move to, and tend to reduce the scope of their wandering and to restrict their conflicts to their immediate neighbors, thus reducing the quality of the assessments they make of the yields of the different populations of the society.

On the other hand, in a population with low density, the agents may take long times to find other agent to fight with, so retarding the decision about whether to change or not their strategy. As these individual decisions affect the total variation of the population at each moment, the global result of such changes takes longer to appear, thus requiring a larger simulation time to stabilize the number of individuals in each population.

Also, the development of the agent-based multilevel simulation model with external oracles able to globally assess the yields of the sub-populations allowed the simulations to provide more faithful results than the ABM that operates without such oracles.

The two final conclusions that we reach are: first, that this work showed the possibility of the development of a systematic study of the interdependencies of ABMs and EBMs when they are used complementarily; second, that the idea of taking EBMs as formal specification of ABMs seems fruitful, as EBMs may then function as yardsticks for the assessment of the quality of the ABMs results; and third, that a simulation model, like the agent-based multilevel model, that adequately combines resources taken from both EBMs and ABMs may provide better simulation results than simulations based on just ABMs.

Acknowledgements: This work was partially supported by CAPES and CNPq.

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